The binary option pricing for AAPL based on Monte-Carlo simulation

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Abstract: Since 2008, binary option has become more accessible and attracted the attention of investors due to the diversity of trading ways. In this paper, Black-Scholes model and Monte Carlo simulations are utilized to simulate the AAPL stock in one year and price the binary option. Besides, we also analyzed the sensitivity of pricing binary options in aspects of payoff, time, strike price and volatility. According to the analysis, the results came out to be surprisingly valid, which indicates that the price of binary option will change in laws. Our limitation is that although it is theoretically correct, the binary option is more like a gambling, which is more comprehensive and riskier than it is expected in our analysis. Moreover, the way to analyze the sensitivity of volatility need to improve to be more valid. Nevertheless, these results could offer investors some suggestions to make profit in the real market and shed light on binary option pricing.

1. Introduction

The first time an option appeared was the “bucket shops” [1]. This barrel shop in the United States in the 1920s was famous for a man named Jesse Livermore. Livermore speculates on the trend of stock prices and predicts their future prices. In the beginning, his company was just a stock options bookmaker, as opposed to anyone who believed that the price of a particular stock might rise or fall. Afterward, his company model achieved great success, and people began to recognize the existence of this form and the options contract market gradually began to form. In 2008, options were finally publicly made available for the first time as tradable assets on CBOE. Meanwhile, the subprime mortgage crisis in the United States triggered one of the worst financial meltdowns in human history in 2008. Investors and economists need more low-risk investment to stimulate the economics of the future. Under this situation, binary options appeared and became more accessible and attracted the attention due to the diversity of trading ways. Black-Scholes model and Monte Carlo model are utilized to simulate the AAPL stock in one year and priced the binary option. Besides, we also analyzed the sensitivity of pricing binary options because of the diversity of trading ways. As mentioned in Ref [2], unlike other options and stocks, the risk of this investment is flexible and easy for investors to handle. However, the binary option is still not mature enough. Thus, we need to investigate binary options to help customers learn sensitivity of pricing-controlling well of investment and anticipate more precisely.

In this research, we will deeply analyze Apple company’s binary option to know how sensitive the binary option market is. Apple’s options markets are relatively huge compared to other option markets. Although the stock price of Apple fluctuates to a certain extent at the beginning of 2021, the overall trend is slowly rising. If Apple’s stock price is slowly rising in the long term, the call option is a good choice for investors. However, in the short term, stock price fluctuations will be more obvious. The Zacks Equity Research argues that Apple’s option implied volatility was much higher [3], which means that within a certain time frame, investors will have a lot of additional buying or selling. In this case, the huge market capital will leave individual investors with huge risks. Nowadays, Apple stock price is hovering between 145 USD. Thus, helping investors learn sensitivity, volatility of option and pricing-controlling well of investment is necessary for them to invest smarter.

For motivation, firstly, starting by anticipating direction and easy to get started, which fits traders who just started to invest, since a binary option contract is a simple derivative contract with only two possible outcomes. Secondly, as mentioned in Ref. [4] it mentions sensing of timing, which means the expiry time can be long or short. Thus, there is a high degree of freedom in trading when to join in and when to get out of the market. Lastly, it is a way to play, offering a playing field [5], whose play field trend is becoming universal and popular. Binary options trading is about managing risks and rewards. It is like a gambling, since one cannot avoid losses, but will win a lot if you follow principles and manage well. When you are involved in this trade, not just to make a quick buck but because you love the game.

In this paper, we analyzed the impact of changes in the variables in different Parameters on the Apple's binary option price. The data and graphs show the fluctuation trend and risk sensitivity of different variables. We first explained the source of data and method and explained the use of each publicity. Subsequently, we discuss the sensitivity analytical results and explain the source and theoretical basis of the chart data, and finally a summary of the full text is presented.

2. Data &Method

2.1 Data

We download daily stock close price of AAPL from 4-Jan-2021 to 26-August-2021 from Yahoo Finance [7]. A market Chart of AAPL is shown in Fig. 1. As illustrated in the Fig. 1 of AAPL in a period of almost 8 months, we can observe that the stock price was basically fluctuating between 120 USD and 150 USD. At the last day in our time scope, which is 26-August-2021, the price is shown as 147.54 USD, which we will consider as the current stock price of AAPL in later analysis.

Fig. 1. Market Chart of AAPL.

2.2 Black-Scholes model

The Black-Scholes (BS) model, also known as the Black-Scholes-Merton (BSM) model, is one of the most important concepts in modern financial theory. This mathematical equation estimates the theoretical value of derivatives and other investment instruments, taking into account the impact of time and other risk factors. It was developed in 1973 and is still regarded as one of the best ways to price option contracts [8].

There are several assumptions in the B-S model, and such assumptions are as follow: First, no dividends are paid during the validity period of the option. Second, the market is random (that is, market trends cannot be predicted). Third, there are no transaction costs for buying options. Fourth, the risk-free interest rate and volatility of the underlying asset are known and constant. Fifth, the return on the underlying asset is log-normally distributed. Sixth, options are European style and can only be exercised when they expire [9].

The formula of B-S model is as follow:

$$C = SN(d_1) - Ke^{-rt}N(d_2) \tag{1}$$

where

$$d_1 = \frac{ln\frac{S}{K}+(r+\frac{\sigma^2}{2})t}{\sigma\sqrt{t}} \tag{2}$$

and

$$d_2 = d_1 - \sigma\sqrt{t} \tag{3}$$

Here, *C* means the price of the call option, *S* means the current stock price of AAPL, *K* means the strike price we set, *r* means the risk-free interest rate, *t* means time to maturity, *σ* means the yearly volatility of the stock price of AAPL, *N* means a normal distribution.

2.3 Monte-Carlo Simulation

A Monte Carlo simulation, also known as Monte Carlo method or multiple probability simulation, is a mathematical technique used to estimate the possible outcomes of uncertain events [10]. Given the inherent variability of the market and the need for financial professionals to evaluate strategies with uncertain results, the idea of applying Monte Carlo simulation to finance naturally emerged [11]. We choose to use Monte Carlo simulation because it can stimulate the stock price of AAPL and it can be realized in Excel.

In this Monte-Carlo simulation, we set the current stock price of AAPL as 147.54 USD, the risk-free rate as 0.15%, time to maturity as 1 year, the yearly volatility of the stock price of AAPL as 25.67%, which is calculated from the historical data (the daily close price, of AAPL from 4-Jan-2021 to 26-August-2021) in Excel, and the dividend as 0.58%, which is searched in the website of Yahoo Finance.

3. Results & Discussions

Tables. I and II are the parameters we set for the project and the data gained through standard simulation. In Table I, there are the parameters we used for our project. To be specific, the stock price and dividends are collected from Apple company's last year's historical data. The volatility numbers are gained from the above equation. All other parameters here in Table II are the numbers set for this project. In Table II, one can see the parameters we gain from using the standard simulation for at time T and time 0.

Table I: The Parameters description

Parameters	
Stock Price	147.54
Strike Price	185
Risk-free Rate	0.15%
Time	1
Volatility	25.67%
Dividends	0.58%
Payoff	80

Table II: The Parameters Gained through Standard Simulation

Average at time T	12.37	
Standard Deviation	28.92	
Sample	10,000	
Standard Error	0.29	
Maximum Error (2nd)	0.58	
At time 0		
Present Value	12.35	
Maximum Error	0.58	
Range	11.77	12.93

Fig. 2 is the payoffs of binary options set in this paper. The X-axis represents the strike price and the Y-axis represents the payoff of the binary option. There are two horizontal lines in Fig. 2, the lower line represents that when the price is lower than 185, which is the value we set, the payoff will be zero. Besides, the upper line represents that when the price is above 185, one will gain 80 payoff, which is also the expected value.

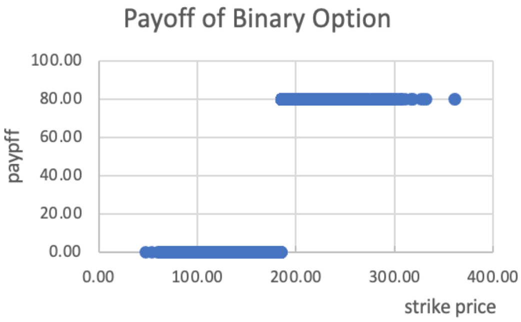

Fig. 2. Payoff of Binary Option.

Fig. 3 presents the Strike price vs. Stock price (only 100 times of 10,000 samples). The X-axis represents the times we conduct the sample and the Y-axis represents the price value. The blue line in Fig. 3 represents the strike price of 185 (the value we set) and the orange line represents the stock price for 100 times out of 10,000 times. Based on this result, one sees that the frequency at which the stock price is higher than the strike price is much lower than the frequency at which the stock price is lower than the strike price.

Fig. 4 depicts the relationship of sensitivity to payoff. The X-axis represents the payoffs and the Y-axis represents the binary option price value. Apparently, the linear relationship is witnessed in the figure. The reason for the linear relationship is that in the situation of all other inputs remains the same except the payoffs, the stock price remains the same, and the value gained will be higher.

Fig. 5 is the relationship of sensitivity to time. The X-axis represents the time in years and the Y-axis represents the binary option price value. As illustrated in Fig. 5, the line firstly increases in the beginning five years and then decreases for the longer maturity time.

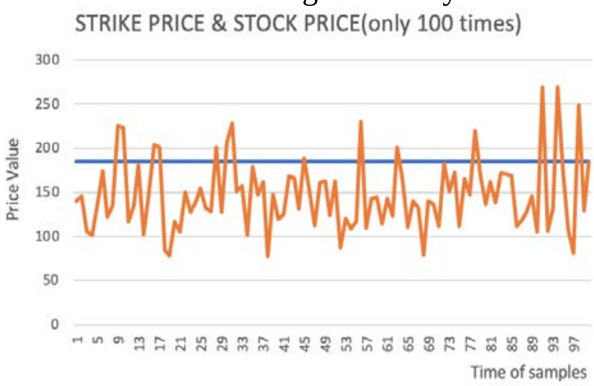

Fig. 3. Strike price vs. Stock price (only 100 times of 10,000 samples).

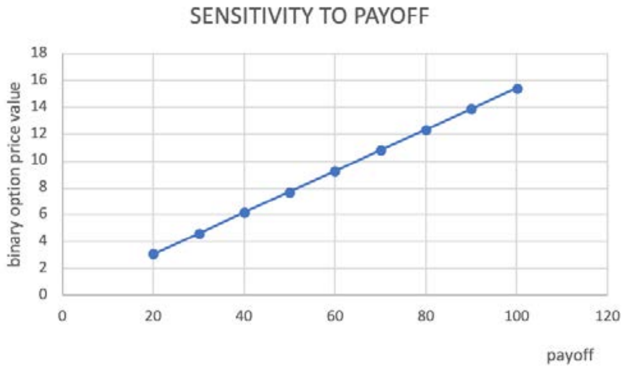

Fig. 4. Sensitivity to Payoff.

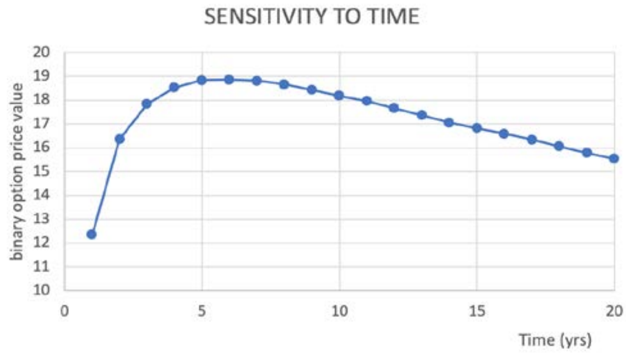

Fig. 5. Sensitivity to Time.

Figs. 6 and 7 illustrate the reason for the performance of the line in Fig. 5 acts (firstly increase within 5 years). The X-axis of both represents the times of sample while their Y-axis represent the price value. Fig. 6 represents the maturity time of 1 year and the ratio of stock price bigger than strike price is 16%. Fig. 7 demonstrates the maturity time of 5 years and the ratio of stock price bigger than strike price is 25%. This means that within 5 years, as time passes, people would have more chances to get their payoffs.

$$S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + z\sigma\sqrt{T}} \tag{4}$$

$$PV = Payoff_{ave}\, e^{-rT} \tag{5}$$

Eqs. (4) and (5) give the reason for the performance of the line in Fig. 5 acts (decreases with longer maturity time). Eq. 4 is the evolution equation of S_T. With time going, T goes up, S_T mathematically drops down. On this basis, the ratio of strike price higher than stock price rises, i.e., the line slows in following years. Eq. 5 is the equation of present value. As time increases, the discount time also becomes longer. Due to the time value, the present value of the binary option decreases.

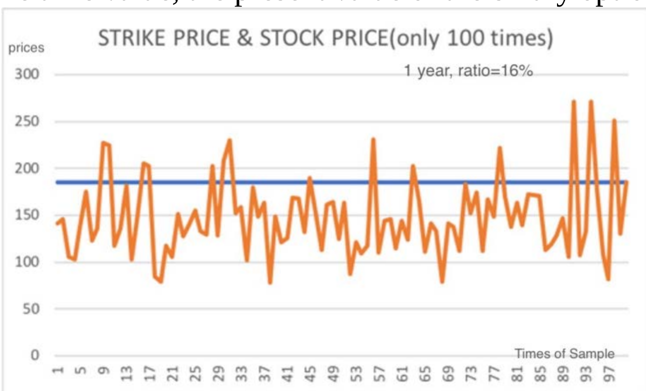

Fig. 6. Strike price vs. Stock price (only 100 times of 10,000 samples), Maturity Time= 1 year, Ratio=16%.

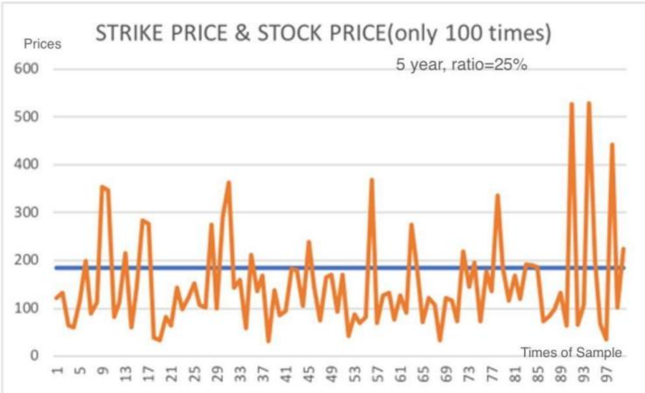

Fig. 7. Strike price vs. Stock price (only 100 times of 10,000 samples), Maturity Time= 5 year, Ratio=25%

Fig. 8 displays the simulation stock prices with different strike prices (different horizontal lines). The X-axis represents the time of sample and the Y-axis represents the strike price value. The fold line in Fig. 8 represents the ratio of the stock price bigger than the strike price, which is the reason why the binary option price will be sensitive to the strike prices. Fig. 9 shows the relationship of sensitivity to strike price. The X-axis represents the strike price value and the Y-axis represents the payoffs. From Figs. 8 and 9, we can see that when the strike price is extremely high, the stock price will hardly be higher than the strike price, and the value we gain would be 0. When the strike price is extremely low, the stock price is always higher than the strike price, the proportion of execution power is extremely high, and the value tends to be the highest payoff, which is 80 we set here.

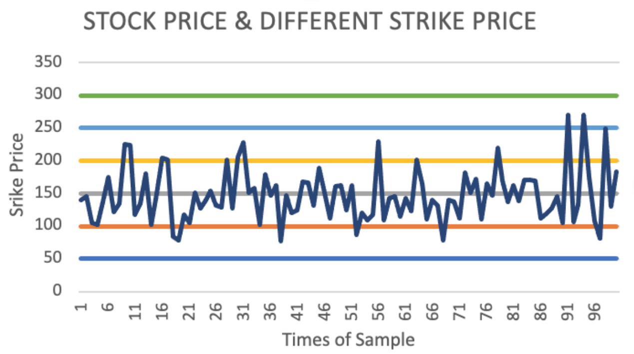

Fig. 8. Simulation Stock Prices with Different Strike Prices.

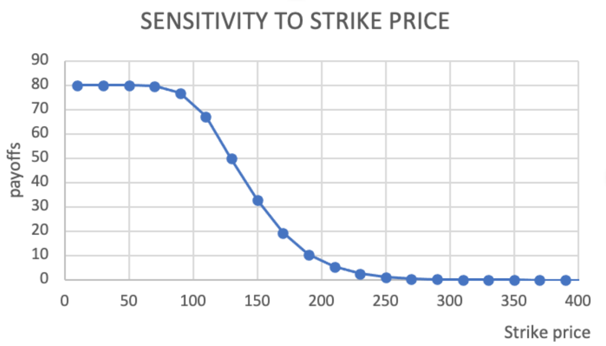

Fig. 9. Sensitivity to Strike Price

Fig.10 exhibits the relationship of sensitivity to volatility when the strike price is smaller than the spot price, where strike price = 100 and the spot price = 147.5 (the value of most recent stock price of Apple). The X-axis of it represents the volatility and the Y-axis represents the value gained. Figs 11-13 demonstrate the reason for the performance of the line in Fig.10 acts (decreases with volatility goes up). These figures each display the situation when the volatility equals 5%, 25% and 80%. The X-axis of all three graphs represent the times of sample and the Y-axis represents the strike price value. According to the results, as the volatility increases, the ratio of stock price bigger than strike price continues to decrease. Thus, the line in Fig. 10 continues to slow down. Based on Eq. (4), with volatility growing up, σ goes up, S_T will mathematically go down. Therefore, the ratio of strike price lower than stock price rises and the value gained goes down.

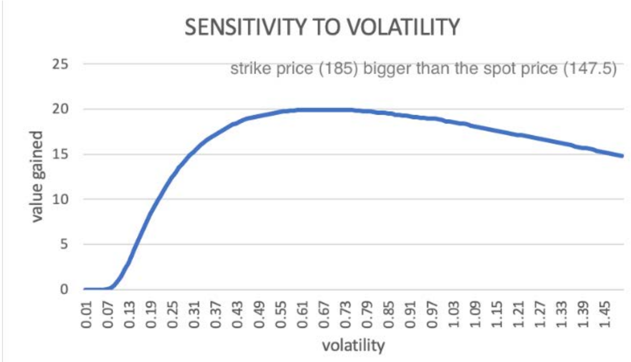

Fig. 10. Sensitivity to Volatility, Strike price(100) smaller than the Spot price(147.5).

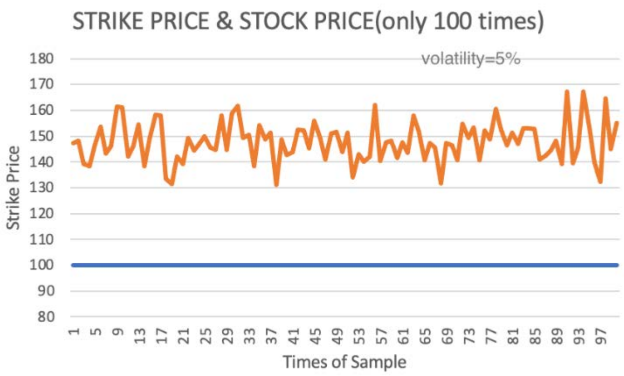

Fig. 11. Strike price vs. Stock price (only 100 times of 10,000 samples), Volatility = 5%.

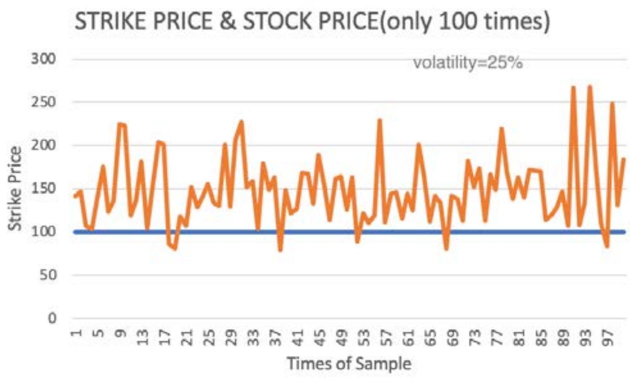

Fig. 12. Strike price vs. Stock price (only 100 times of 10,000 samples), Volatility = 25%.

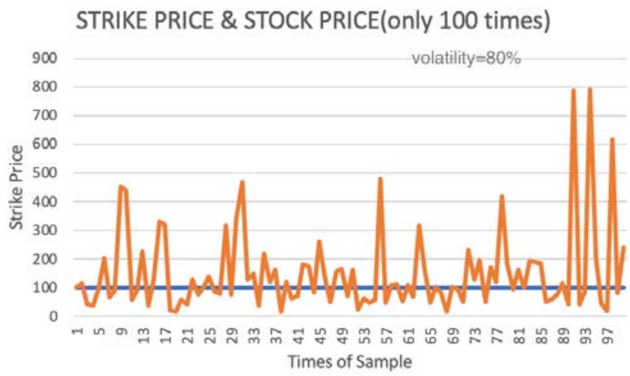

Fig. 13. Strike price vs. Stock price (only 100 times of 10,000 samples), Volatility = 80%.

Fig. 14 gives the relationship of sensitivity to volatility when the strike price is bigger than the spot price, where strike price = 185 and the spot price = 147.5 (the value of most recent stock price of Apple). The X-axis of Fig. 14 represents the volatility and the Y-axis represents the value gained. Figs.15-17 show the reason for the performance of the line in Fig.14 acts (increases with volatility goes up). These charts each display the situation when the volatility equals 5%, 25% and 80%. The X-axis of all three graphs represent the times of sample and the Y-axis represents the strike price value. Based on the analysis, as the volatility increases, the ratio of stock price bigger than strike price continues to increase. Thus, the line in Fig. 14 continues to go up.

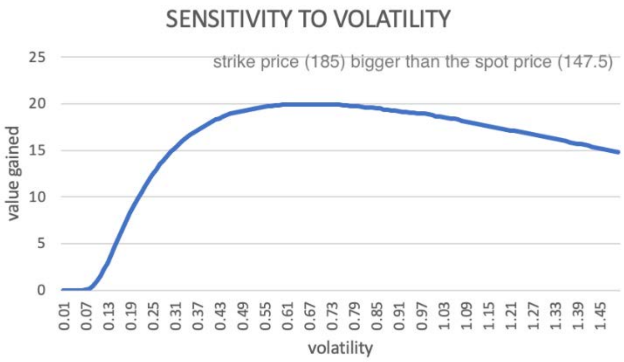

Fig. 14. Sensitivity to Volatility, Strike price(185) bigger than the Spot price(147.5).

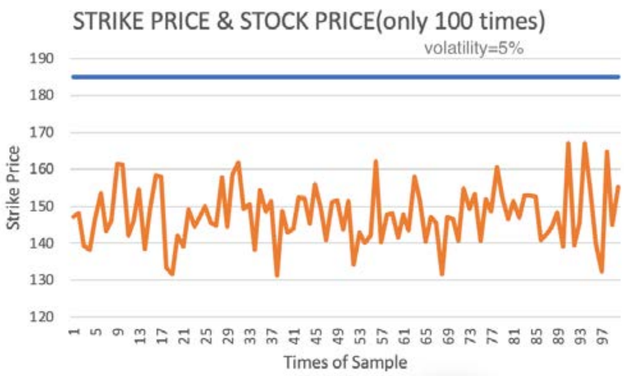

Fig. 15. Strike price vs. Stock price (only 100 times of 10,000 samples), Volatility = 5%.

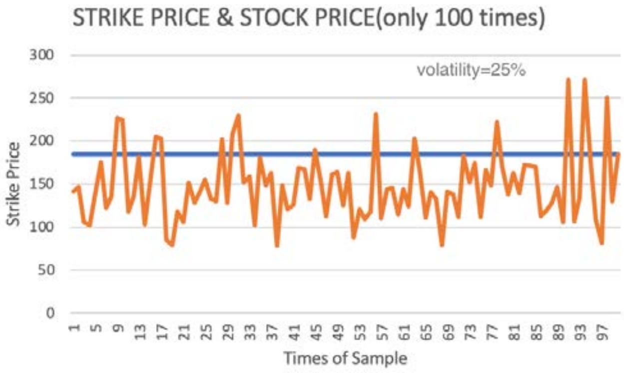

Fig. 16. Strike price vs. Stock price (only 100 times of 10,000 samples), Volatility = 25%.

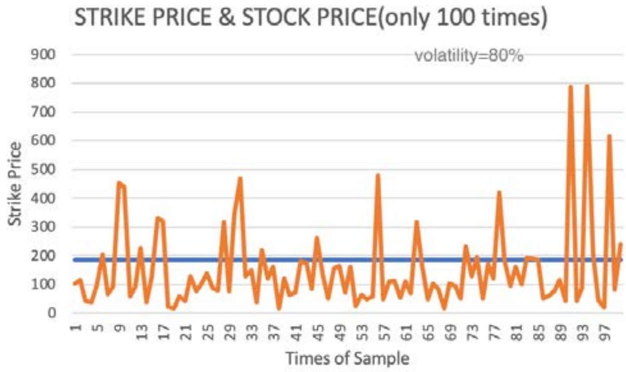

Fig. 17. Strike price vs. Stock price (only 100 times of 10,000 samples), Volatility = 80%.

Although we stimulated the stock price and analyzed the sensitivity of the option price to some of the parameters quite correctively, there are still some limitations in our research. First, the B-S pricing model and the Monte Carlo simulation have some pragmatic problems, although they are valid theoretically. Second, the Binary option is more likely to be a gambling. In the real world, the binary option is more comprehensive and riskier than showed in the analysis. The investors should pay more attention when they buy it. Third, considering the sensitivity, we mainly focused on the relationship of sensitivity to the payoff, time, strike price and volatility of the Binary option Pricing and gained related outcomes. These outcomes may give investors some ideas of binary options. However, we believe that these research aspects are not comprehensive enough. For future improvement, we may consider the option price’s sensitivity to the volatility of the strike price.

4. Conclusion

In summary, we investigate the binary option pricing for AAPL based on B-S model and Monte Carlo model. Specifically, we set the current price as 147.54 dollars, which is the stock price of the last day in our time scope. Through the historical data, we calculated the volatility. With the strike price as 185 USD, the payoff as 80 dollars, the risk-free risk as 0.15%, maturity time as one year and the dividends as 0.58%, we calculated the price of the binary option as 12.35 USD. Besides, the sensitivity analysis of the option price are carried out. According to the analysis, the binary option price is sensitive to payoff, the maturity time, the strike price and the volatility. Nevertheless, our results have certain limitations though it is theoretically correct. Similar to a gamble, the real binary option in the market is more complex and riskier. More importantly, the method of analyzing volatility sensitivity is not clear enough. Nevertheless, our results shed light on future binary option implementation in real market anyway.

5. Conflict of Interest

The authors declare no conflict of interest.

6. Author Contributions

These authors contributed equally.

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